

# Contact-dependent balance stability of biped systems

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## Introduction

Instinctively, for tasks involving multiple time-varying contacts interactions (e.g., walking, climbing stairs, etc.) humans plan their predictable contact configurations such that balance stability during the given task is not compromised (Padois et al., 2016). On the other hand, designing balance controllers that allow biped robots to exploit the available contact interactions, while predicting their effects on the system’s balance stability, is still a challenge.

In this work, a novel computational approach (Mummolo et al., 2017) is extended to investigate the effects of various contact configurations on the balancing capabilities of constrained biped systems. A balance stability region is identified in the center of mass (COM) state space that includes all states (position and velocity) from which the biped system can be balanced while satisfying physical constraints and maintaining the specified contact configuration. The stability region boundary is constructed via an optimization-based algorithm and provides a contact-dependent threshold between balanced and falling states for the given system. A COM state outside of the stability region boundary represents the sufficient condition for a falling state, from which a change in the specified contact configuration is inevitable.

Experimental trajectories of robotic and human walking are analyzed relative to their corresponding contact-dependent stability regions, to gain insights on the different balance stability strategies between robot and human in various phases of the walking motion.

## Methods

The relative notions of balanced and falling states for a biped system are herein interpreted with respect to a specified contact configuration (Fig. 1). For instance, let’s consider a biped system balancing on a single foot (Fig. 1; top-left). If, at a given COM position  $\bar{\mathbf{r}} = (\bar{x}, \bar{y}, \bar{z})$ , the COM velocity is within a certain threshold  $\dot{\mathbf{r}}_{SS}^{\text{lim}} = (\dot{x}_{SS}^{\text{lim}}, \dot{y}_{SS}^{\text{lim}}, \dot{z}_{SS}^{\text{lim}})$ , then balance can be maintained while satisfying necessary physical

constraints and preserving the single support (SS) contact configuration. In this case, the system is said to be balanced with respect to SS, e.g., the system can reach a final static equilibrium (Koolen et al., 2012) without altering its contacts, but simply enabled by its initial conditions and available actuation. Conversely, if the velocity perturbations at the COM surpass the threshold  $\dot{\mathbf{r}}_{SS}^{\text{lim}}$ , then the biped won’t be able to stop unless SS contact is altered. In this case, the initial COM state is said to be falling with respect to the intended SS contact (Fig. 1), and will end up in a fallen state relative to SS, e.g., double support (DS). The threshold  $(\bar{\mathbf{r}}, \dot{\mathbf{r}}_{SS}^{\text{lim}})$  represents the system’s balance stability boundary, which is dependent on the specified contact configuration.

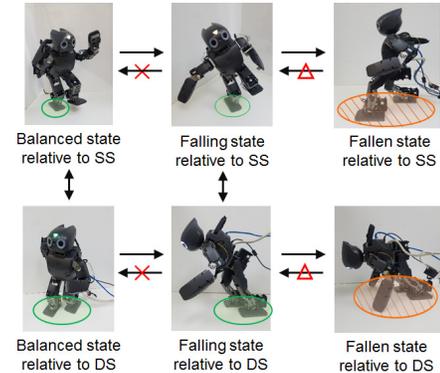


Figure 1. The notion of balance stability states for a biped system are dependent on the specified contact configuration.

For a planar biped model in the  $(X, Y)$  sagittal plane, when the COM velocity threshold is searched along the  $x$ -direction, the balance stability boundary for a specified contact configuration  $(\cdot)$  is the set of points:

$$S_{(\cdot)} = \{(\bar{x}, \bar{y}, \dot{x}_{(\cdot)}^{\text{lim}}, \dot{y}_{(\cdot)}^{\text{lim}}) \mid \dot{x}_{(\cdot)}^{\text{lim}} = \dot{x}_{(\cdot)}^{\text{Max}}, \dot{x}_{(\cdot)}^{\text{Min}}\} \quad (1)$$

where  $\dot{x}_{(\cdot)}^{\text{Max}}$  and  $\dot{x}_{(\cdot)}^{\text{Min}}$  are the COM velocity extrema along  $x$ , evaluated at a the COM position through constrained optimization. Using an iterative algorithm, the maximum allowable COM velocity perturbations are evaluated at discretized points of the

COM workspace, by imposing  $(\bar{x}, \bar{y}) = (x_k, y_j)$  (Fig. 2). Additional constraints include center of pressure bounds for each contact area, positive normal contact forces, friction constraints, joint angle, velocity, and torque limits, final equilibrium at an arbitrary home configuration, and constraints to ensure that the specified contact configuration remains unaltered.

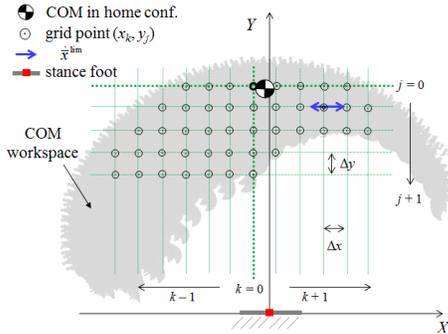


Figure 2. Biped system COM workspace (e.g., in SS), discretized via grid points at which the COM velocity extrema are evaluated.

Since this work is extended to the case of multi-contact conditions, the following challenges are addressed: (1) to formulate a constrained multi-body dynamic model for biped systems in which the indeterminacy between the motion, control, and ground reactions is resolved in a physically consistent manner; (2) to design a numerically efficient and kinematics-consistent method for evaluating and discretizing the complete COM workspace in SS and DS contact configurations; (3) to establish a systematic optimization-based algorithm for the construction of balance stability boundaries that are dependent on the specified contact.

## Results and Discussion

The contact-dependent balance stability boundary are evaluated for the biped robot DarwIn-OP and analyzed with respect to its experimental walking trajectory. The stability region in DS contact configuration is smaller than that of SS (Fig. 3). Hence, during SS the robot can recover from larger COM velocity perturbations in the  $x$  direction, as opposed to DS, while simultaneously ensuring that the contact status between robot parts and the environment is unaltered.

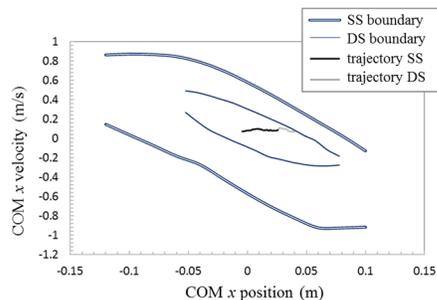


Figure 3. Results of the biped robot stability boundaries and one-step trajectory in the state space of COM  $x$  position and velocities.

Every state in the one-step walking trajectories results balanced with respect to SS and DS. This implies that the balance controller available for the robot platform (DarwIn-OP, ROBOTIS) is designed to generate walking motions that are very “conservative” (i.e., far from stability boundaries, except for late DS phase).

The stability boundary results and the experimental one-step trajectory are also shown for a biped model based on a real human subject (Fig. 4), for which a more refined gait segmentation is chosen (SS1, flat foot; SS2, toe contact; DS). The human balancing strategy during SS1 is in contrast to what observed above for the robot SS, since great part of the COM states in SS1 phase lies outside of the corresponding stability boundary. These results reveal the well-known basic principle that natural human walking is characterized by series of “controlled forward falls”, from one foot stance to the next foot contact.

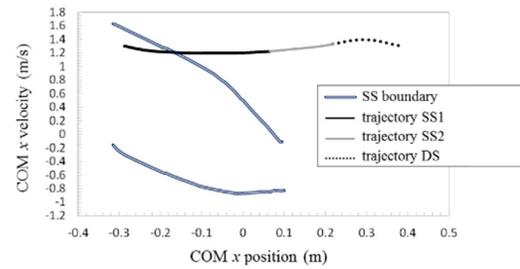


Figure 4. Results of the human subject stability boundaries in the state space of COM  $x$  position and velocities, during SS1. Boundary results for SS2 and DS are part of on-going work.

For general robotic gait control applications, the balance stability region provides a “map” of balanced states relative to a given contact configuration. This map could be a useful reference for the (re-)design of system- and contact-specific balance controllers, whose performance region can be analyzed in the state space, relative to the contact-dependent stability regions evaluated beforehand. This could provide, for instance, guidelines for more advanced balance controller for robots that are able to perform a less conservative and more efficient passive dynamic walking and achieve more human-like efficient locomotion.

## References

- V. Padois, S. Ivaldi, J. Babič, M. Mistry, J. Peters and F. Nori, "Whole-body multi-contact motion in humans and humanoids: Advances of the CoDyCo European project," *Robotics and Autonomous Systems*, 2016. doi: 10.1016/j.robot.2016.08.017.
- C. Mummolo, L. Mangialardi and J. H. Kim, "Numerical Estimation of Balanced and Falling States for Constrained Legged Systems," *Journal of Nonlinear Science*, published online, pp. 1-33, 2017. doi: 10.1007/s00332-016-9353-2.
- Koolen, T., et al. (2012). Capturability-based analysis and control of legged locomotion, Part 1: Theory and application to three simple gait models. In: *The Intl. J. of Robotics Research*, vol. 31, no. 9, pp. 1094-1113, 2012. doi: 10.1177/0278364912452673.